

Algebra & Combinatorics in CIRCA

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People: Staff

- Dr Martyn Quick (Head of Group)
- Prof Nik Ruskuc (Head of Pure Mathematics)
- Dr Robert Gray
- Dr Sophie Huczynska (mat. leave)
- Dr James Mitchell
- Dr Max Neunhoeffler
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People: Students

- Nabilah Abughazalah
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- Hannah Coutts
- Simon Craik
- Andreas Distler
- Jillian Ferguson
- Arthur Geddes
- Victor Maltcev
- Nina Menezes
- Jonathan McDougall-Bagnall
- Yann Perese
- Markus Pfeiffer
- Claire Pollard

Algebra & Combinatorics

- **Algebra**
 - sets with operations
 - language and methods for: symmetry, similarity, correspondences, generation, ...
- **Combinatorics**
 - sets with relations
 - static structures, relationships

Algebra & Combinatorics

Other possible divisions:

- Finite vs. infinite
- Constructive vs. existential
- Computational vs. theoretical

These run across all areas of activity

Promote interplay

Algebra

- **Groups:** finite, general infinite, profinite, finitely presented, representation theory, automorphisms
- **Semigroups:** combinatorial theory, infinite transformations, actions, endomorphisms
- **General/universal algebra:** constructions, generators and relations, finiteness conditions

Combinatorics

- **Pattern classes of permutations:** structure, constructions, enumerations, algorithms, decidability
- **Relational structures:** parallels universal algebra strand - consider the same problem for different structures
- **Infinite structures:** random graph and variations

Flagship External Collaborations

- Scotland: Glasgow, Heriot-Watt (combinatorial semigroup theory)
- UK: Warwick (Groups, CGT), Leicester (automata in algebra), Manchester (combinatorial semigroup theory), Leeds (infinite combinatorics, graph endomorphisms)
- World: Otago, NZ (pattern classes); Aachen, Germany (CGT, representation theory); Wroclaw, Poland (infinite semigroups and groups of transformations); Lisbon, Porto, Portugal (semigroups), Sydney, Australia (CGT, comp. algebra)

Points of Interaction: Current

Decidability

- Semigroups -- the entry of undecidability into algebra (encoding Turing machines and other undecidable problems)
- Decidability in combinatorics -- a new development pioneered by us

Points of Interaction: Current

Algebraic algorithms

- Groups (permutations, matrices, finitely presented, representation theory)
- Semigroups (structure, enumeration, endomorphisms, finitely presented)

Points of Interaction: Current

Automata

- Simplest models of computation
- Semigroup actions in disguise
- Use them to define semigroups
- Algorithmically determine properties from automata
- Automatic structures, FA presentability, automata semigroups

Points of Interaction: Current

Computations in pattern classes

- Encodings over a finite alphabet
- Associated algorithms
- Applications to enumeration, growth rates, sorting networks, etc.

Points of Interaction: Future

Actions

- Absolutely central for both theory and computations
- Unique expertise profile: groups/ semigroups/ linear, finite/ infinite, theory/ applications
- Time to develop an overarching general theory + computational applications?

Points of Interaction: Future

Algorithmic treatment of 'large infinite' structures

- Use automata to approach algorithmically non-finitely-generated, or even uncountable structures
- Test case: Automorphism group and endomorphism monoid of infinite regular rooted tree via automata groups and semigroups

Points of Interaction: Future

Universal Algebra/ Model Theory

- Work with terms/ trees instead of words/ lists
- Curious links with CSP and P/NP
- ‘Computational Combinatorics’ - extend our methods from pattern classes - a promising ‘practical’ area

Conclusion: Methods vs. Language

- Each of the above topics represents a set of problems and methods for solving them
- At the same time: a language
- Today: emphasis on methods and results
- We mustn't forget the language -- it is the foundation for tomorrow's methods and results