Constraints

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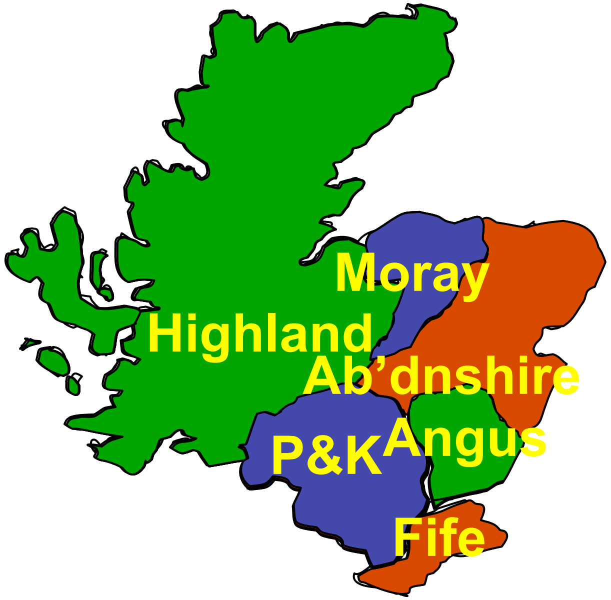
Constraints: A Natural Means of Knowledge Representation



- $x + y = 30$
- Adjacent countries on map cannot be coloured same.
- The helicopter can carry one passenger.
- Maths class must be scheduled between 9 - 11 am.

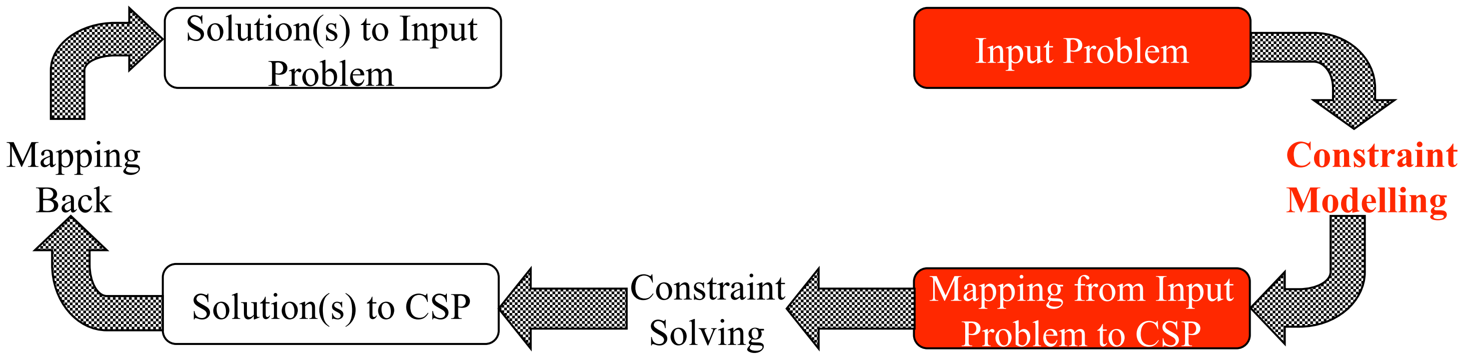
Constraint Solving

- Offers an efficient means of finding solutions to combinatorial problems.
  - E.g. Planning, Scheduling, Timetabling…
- A **constraint model** is a description of a combinatorial problem in a format suitable for input to a **constraint solver**.
- Constraint solver searches for solutions to the problem automatically.

Who Cares about Constraint Solving?

- **Many** important industrial applications.
- Visit www.ilog.com to see their customer list, including:
  - AT&T, Deutsche Telekom, France Telecom.
  - United Airlines, Southwest Airlines.
  - Compaq, Nissan, Visa.
- Google, Microsoft recruits CP people
- CISCO funded development of CP tool
- IBM bought ILOG

Constraint Modelling & Solving



- A constraint model is a description of a combinatorial problem in terms of a **constraint satisfaction problem** (CSP).
  - The features of a given problem are mapped onto the features of a CSP.
  - Our tool **Tailor** can help with modelling.

Constraint Satisfaction Problems

- Given:
  - A finite set of **decision variables**.
  - For each decision variable, a finite **domain** of potential values.
  - A finite set of **constraints** on the decision variables.
- Find:
  - An assignment of values to variables such that all constraints are satisfied.

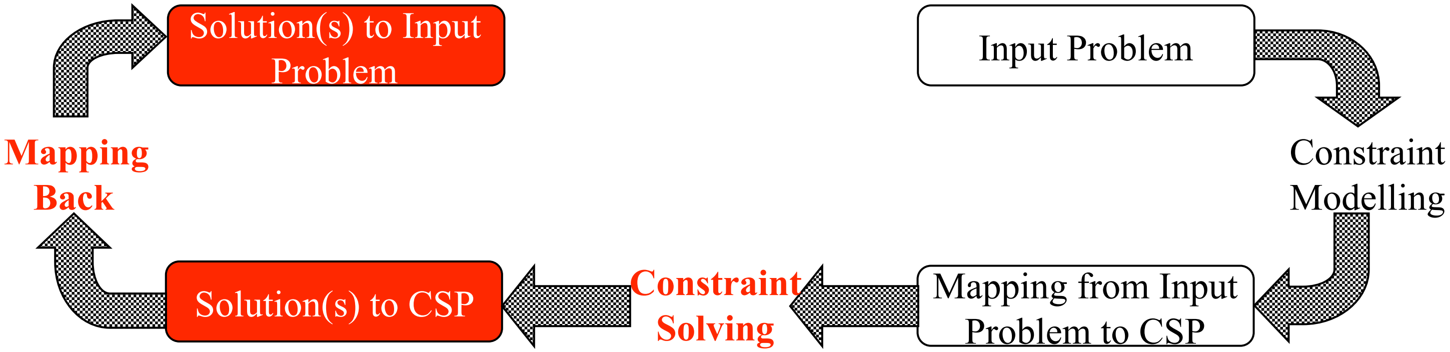
Decision Variables & Domains

- A decision variable corresponds to a **choice** that must be made in solving a problem.
  - E.g. university timetabling: we must decide the time & venue for each lecture.
- **Values** in the domain of a decision variable correspond to the various **options** for this choice.
  - E.g. lecture time: 9am, 10am, …
  - E.g. lecture venue: Physics A, Maths A, …
- A decision variable is **assigned** a value from its domain.
  - Equivalently, the choice associated with that variable is made.

Constraints

- A constraint specifies allowed/disallowed combinations of assignments:
  - No pair of lectures can share the same time and venue.
  - No pair of lectures given by the same lecturer can share the same time.

Constraint Modelling & Solving



- The CSP is input to a constraint solver (e.g. our own **Minion** solver), which produces a solution (or solutions).
- The model is used to map the solution(s) back onto the original problem.

Example: Sudoku

- Example taken from
  - “Sudoku as a Constraint Problem”,
    - by Helmut Simonis.

The Sudoku Problem

	2	6				8	1	
3			7		8			6
4				5				7
	5		1		7		9	
		3	9		5	1		
	4		3		2		5	
1				3				2
5			2		4			9
	3	8				4	6	

- Given: a 9×9 grid, with some entries blank, some containing a digit.
- Find: a complete grid.

The Sudoku Problem: Constraints

	2	6				8	1	
3			7		8			6
4				5				7
	5		1		7		9	
		3	9		5	1		
	4		3		2		5	
1				3				2
5			2		4			9
	3	8				4	6	

- Such that:
  - On any row, all entries are distinct.

The Sudoku Problem: Constraints

	2	6				8	1	
3			7		8			6
4				5				7
	5		1		7		9	
		3	9		5	1		
	4		3		2		5	
1				3				2
5			2		4			9
	3	8				4	6	

- Such that:
  - On any column, all entries are distinct.

The Sudoku Problem: Constraints

	2	6				8	1	
3			7		8			6
4				5				7
	5		1		7		9	
		3	9		5	1		
	4		3		2		5	
1				3				2
5			2		4			9
	3	8				4	6	

- Such that:
  - These (the red & white) 3 × 3 squares contain distinct entries.

Sudoku as a CSP

	2	6				8	1	
3			7		8			6
4				5				7
	5		1		7		9	
		3	9		5	1		
	4		3		2		5	
1				3				2
5			2		4			9
	3	8				4	6	

- 81 variables, one for each grid entry.
- Domain: {1, …, 9}
  - For simplicity we'll assume that pre-filled entries are represented by variables with singleton domains.
- All-different constraints on rows, cols, 3 × 3 squares.

Sudoku as a CSP

{1,2,3,4,5, 6,7,8,9}	2	6	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	8	1	{1,2,3,4,5, 6,7,8,9}
3	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	7	{1,2,3,4,5, 6,7,8,9}	8	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	6
4	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	5	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	7
{1,2,3,4,5, 6,7,8,9}	5	{1,2,3,4,5, 6,7,8,9}	1	{1,2,3,4,5, 6,7,8,9}	7	{1,2,3,4,5, 6,7,8,9}	9	{1,2,3,4,5, 6,7,8,9}
{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	3	9	{1,2,3,4,5, 6,7,8,9}	5	1	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}
{1,2,3,4,5, 6,7,8,9}	4	{1,2,3,4,5, 6,7,8,9}	3	{1,2,3,4,5, 6,7,8,9}	2	{1,2,3,4,5, 6,7,8,9}	5	{1,2,3,4,5, 6,7,8,9}
1	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	3	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	2
5	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	2	{1,2,3,4,5, 6,7,8,9}	4	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	9
{1,2,3,4,5, 6,7,8,9}	3	8	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	4	6	{1,2,3,4,5, 6,7,8,9}

Sudoku as a CSP

{1,2,3,4,5, 6,7,8,9}	2	6	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	8	1	{1,2,3,4,5, 6,7,8,9}
3	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	7	{1,2,3,4,5, 6,7,8,9}	8	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	6
4	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	5	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	7
{1,2,3,4,5, 6,7,8,9}	5	{1,2,3,4,5, 6,7,8,9}	1	{1,2,3,4,5, 6,7,8,9}	7	{1,2,3,4,5, 6,7,8,9}	9	{1,2,3,4,5, 6,7,8,9}
{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	3		6,7,8,9}			5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}
{1,2,3,4,5, 6,7,8,9}	4	{1,2,3,4,5, 6,7,8,9}	3	{1,2,3,4,5, 6,7,8,9}	2	{1,2,3,4,5, 6,7,8,9}	5	{1,2,3,4,5, 6,7,8,9}
1	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	3	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	2
5	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	2	{1,2,3,4,5, 6,7,8,9}	4	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	9
{1,2,3,4,5, 6,7,8,9}	3	8	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	4	6	{1,2,3,4,5, 6,7,8,9}

Propagate AllDiff on 3 × 3 square.

Sudoku as a CSP

{1,5,7,8, 9}	2	6	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	8	1	{1,2,3,4,5, 6,7,8,9}
3	{1,5,7,8, 9}	{1,5,7,8, 9}	7	{1,2,3,4,5, 6,7,8,9}	8	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	6
4	{1,5,7,8, 9}	{1,5,7,8, 9}	{1,2,3,4,5, 6,7,8,9}	5	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	7
{1,2,3,4,5, 6,7,8,9}	5	{1,2,3,4,5, 6,7,8,9}	1	{1,2,3,4,5, 6,7,8,9}	7	{1,2,3,4,5, 6,7,8,9}	9	{1,2,3,4,5, 6,7,8,9}
{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	3	[illegible]	6,7,8,9}	[illegible]	[illegible]	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}
{1,2,3,4,5, 6,7,8,9}	4	{1,2,3,4,5, 6,7,8,9}	3	{1,2,3,4,5, 6,7,8,9}	2	{1,2,3,4,5, 6,7,8,9}	5	{1,2,3,4,5, 6,7,8,9}
1	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	3	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	2
5	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	2	{1,2,3,4,5, 6,7,8,9}	4	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	9
{1,2,3,4,5, 6,7,8,9}	3	8	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	4	6	{1,2,3,4,5, 6,7,8,9}

Propagate AllDiff on row 1.

Sudoku as a CSP

{5,7,9}	2	6	{3,4,5,7, 9}	{3,4,5,7, 9}	{3,4,5,7, 9}	8	1	{3,4,5,7, 9}
3	{1,5,7,8, 9}	{1,5,7,8, 9}	7	{1,2,3,4,5, 6,7,8,9}	8	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	6
4	{1,5,7,8, 9}	{1,5,7,8, 9}	{1,2,3,4,5, 6,7,8,9}	5	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	7
{1,2,3,4,5, 6,7,8,9}	5	{1,2,3,4,5, 6,7,8,9}	1	{1,2,3,4,5, 6,7,8,9}	7	{1,2,3,4,5, 6,7,8,9}	9	{1,2,3,4,5, 6,7,8,9}
{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	3	[illegible]	6,7,8,9}	[illegible]	[illegible]	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}
{1,2,3,4,5, 6,7,8,9}	4	{1,2,3,4,5, 6,7,8,9}	3	{1,2,3,4,5, 6,7,8,9}	2	{1,2,3,4,5, 6,7,8,9}	5	{1,2,3,4,5, 6,7,8,9}
1	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	3	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	2
5	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	2	{1,2,3,4,5, 6,7,8,9}	4	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	9
{1,2,3,4,5, 6,7,8,9}	3	8	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	4	6	{1,2,3,4,5, 6,7,8,9}

Propagate AllDiff on col 1.

Sudoku as a CSP

{7,9}	2	6	{3,4,5,7, 9}	{3,4,5,7, 9}	{3,4,5,7, 9}	8	1	{3,4,5,7, 9}
3	{1,5,7,8, 9}	{1,5,7,8, 9}	7	{1,2,3,4,5, 6,7,8,9}	8	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	6
4	{1,5,7,8, 9}	{1,5,7,8, 9}	{1,2,3,4,5, 6,7,8,9}	5	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	7
{2,6,7,8, 9}	5	{1,2,3,4,5, [illegible]	1	{1,2,3,4,5, [illegible]	7	{1,2,3,4,5, [illegible]	9	{1,2,3,4,5, 6,7,8,9}
{2,6,7,8, 9}	{1,2,3,4,5, 6,7,8,9}	[illegible]		[illegible] 6,7,8,9}			[illegible] 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}
{2,6,7,8, 9}	4	{1,2,3,4,5,		{1,2,3,4,5,		{1,2,3,4,5,		{1,2,3,4,5,
1	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	3	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	2
5	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	2	{1,2,3,4,5, 6,7,8,9}	4	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	9
{2,6,7,8, 9}	3	8	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	4	6	{1,2,3,4,5, 6,7,8,9}

Propagate AllDiff on 3 × 3 square

Can you see why 3, 4, 9 can be removed?

Sudoku as a CSP

{7,9}	2	6	{3,4,9}	{3,4,9}	{3,4,9}	8	1	{3,4,5,7, 9}
3	{1,5,7,8, 9}	{1,5,7,8, 9}	7	{1,2,6}	8	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	6
4	{1,5,7,8, 9}	{1,5,7,8, 9}	{1,2,6}	5	{1,2,6}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	7
{2,6,7,8, 9}	5	{1,2,3,4,5, 6,7,8,9}	1	{1,2,3,4,5, 6,7,8,9}	7	{1,2,3,4,5, 6,7,8,9}	9	{1,2,3,4,5, 6,7,8,9}
{2,6,7,8, 9}	{1,2,3,4,5, 6,7,8,9}	3	[illegible]	6,7,8,9}	[illegible]	[illegible]	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}
{2,6,7,8, 9}	4	{1,2,3 6,7,8	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	{1,2,3,4,5, 6,7,8,9}
1	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	3	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	2
5	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	2	{1,2,3,4,5, 6,7,8,9}	4	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	9
{2,6,7,8, 9}	3	8	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	4	6	{1,2,3,4,5, 6,7,8,9}

Propagate AllDiff on row 1

Again 3, 4, 9 can be removed

Sudoku as a CSP

{7}	2	6	{3,4,9}	{3,4,9}	{3,4,9}	8	1	{5}
3	{1,5,7,8, 9}	{1,5,7,8, 9}	7	{1,2,6}	8	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	6
4	{1,5,7,8, 9}	{1,5,7,8, 9}	{1,2,6}	5	{1,2,6}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	7
{2,6,7,8, 9}	5	{1,2,3,4,5, 6,7,8,9}	1	{1,2,3,4,5, 6,7,8,9}	7	{1,2,3,4,5, 6,7,8,9}	9	{1,2,3,4,5, 6,7,8,9}
{2,6,7,8, 9}	{1,2,3,4,5, 6,7,8,9}	3	[illegible]	6,7,8,9}	[illegible]	[illegible]	6,7,8,9}	{1,2,3,4,5, 6,7,8,9}
{2,6,7,8, 9}	4	{1,2,3,4,5, 6,7,8,9}	3	{1,2,3,4,5, 6,7,8,9}	2	{1,2,3,4,5, 6,7,8,9}	5	{1,2,3,4,5, 6,7,8,9}
1	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	3	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	2
5	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	2	{1,2,3,4,5, 6,7,8,9}	4	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	9
{2,6,7,8, 9}	3	8	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	4	6	{1,2,3,4,5, 6,7,8,9}

Propagate AllDiff on 3 × 3 square

Sudoku as a CSP

{7}	2	6	{3,4,9}	{3,4,9}	{3,4,9}	8	1	{5}
3	{1,5,7,8, 9}	{1,5,7,8, 9}	7	{1,2,6}	8	{2,3,4,9}	{2,3,4,9}	6
4	{1,5,7,8, 9}	{1,5,7,8, 9}	{1,2,6}	5	{1,2,6}	{2,3,4,9}	{2,3,4,9}	7
{2,6,7,8, 9}	5	{1,2,3,4,5, 6,7,8	1	{1,2,3,4,5,	7	{1,2,3,4,5,	9	{1,2,3,4,5, 6,7,8,9}
{2,6,7,8, 9}	{1,2,3,4,5, 6,7,8,9}	3	[illegible]	6,7,8,9}	[illegible]	[illegible]	6,7,8,9}	{1,2,3,4,5, 6,7,8,9}
{2,6,7,8, 9}	4	{1,2,3,4,5, 6,7,8,9}	3	{1,2,3,4,5, 6,7,8,9}	2	{1,2,3,4,5, 6,7,8,9}	5	{1,2,3,4,5, 6,7,8,9}
1	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	3	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	2
5	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	2	{1,2,3,4,5, 6,7,8,9}	4	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	9
{2,6,7,8, 9}	3	8	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	4	6	{1,2,3,4,5, 6,7,8,9}

Propagate AllDiff on 3 × 3 square

Sudoku as a CSP

{7}	2	6	{3,4,9}	{3,4,9}	{3,4,9}	8	1	{5}
3	{1,5,8,9}	{1,5,8,9}	7	{1,2,6}	8	{2,3,4,9}	{2,3,4,9}	6
4	{1,5,8,9}	{1,5,8,9}	{1,2,6}	5	{1,2,6}	{2,3,4,9}	{2,3,4,9}	7
{2,6,7,8, 9}	5	{1,2,3,4,5, 6,7,8,9}	1	{1,2,3,4,5, 6,7,8,9}	7	{1,2,3,4,5, 6,7,8,9}	9	{1,2,3,4,5, 6,7,8,9}
{2,6,7,8, 9}	{1,2,3,4,5, 6,7,8,9}	3	[illegible]	6,7,8,9}	[illegible]	[illegible]	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}
{2,6,7,8, 9}	4	{1,2,3,4,5, 6,7,8,9}	3	{1,2,3,4,5, 6,7,8,9}	2	{1,2,3,4,5, 6,7,8,9}	5	{1,2,3,4,5, 6,7,8,9}
1	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	3	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	2
5	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	2	{1,2,3,4,5, 6,7,8,9}	4	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	9
{2,6,7,8, 9}	3	8	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	4	6	{1,2,3,4,5, 6,7,8,9}

Propagate AllDiff on row 2

Sudoku as a CSP

{7}	2	6	{3,4,9}	{3,4,9}	{3,4,9}	8	1	{5}
3	{1,5,9}	{1,5,9}	7	{1,2}	8	{2,4,9}	{2,4,9}	6
4	{1,5,8,9}	{1,5,8,9}	{1,2,6}	5	{1,2,6}	{2,3,4,9}	{2,3,4,9}	7
{2,6,7,8, 9}	5	{1,2,3,4,5, 6,7,8,9}	1	{1,2,3,4,5, 6,7,8,9}	7	{1,2,3,4,5, 6,7,8,9}	9	{1,2,3,4,5, 6,7,8,9}
{2,6,7,8, 9}	{1,2,3,4,5, 6,7,8,9}	3	[illegible]	6,7,8,9}	[illegible]	[illegible]	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}
{2,6,7,8, 9}	4	{1,2,3,4,5, 6,7,8,9}	3	{1,2,3,4,5, 6,7,8,9}	2	{1,2,3,4,5, 6,7,8,9}	5	{1,2,3,4,5, 6,7,8,9}
1	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	3	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	2
5	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	2	{1,2,3,4,5, 6,7,8,9}	4	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	9
{2,6,7,8, 9}	3	8	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	4	6	{1,2,3,4,5, 6,7,8,9}

Propagate AllDiff on row 3

Sudoku as a CSP

{7}	2	6	{3,4,9}	{3,4,9}	{3,4,9}	8	1	{5}
3	{1,5,9}	{1,5,9}	7	{1,2}	8	{2,4,9}	{2,4,9}	6
4	{1,8,9}	{1,8,9}	{1,2,6}	5	{1,2,6}	{2,3,9}	{2,3,9}	7
{2,6,7,8, 9}	5	{1,2,3,4,5, 6,7,8,9}	1	{1,2,3,4,5, 6,7,8,9}	7	{1,2,3,4,5, 6,7,8,9}	9	{1,2,3,4,5, 6,7,8,9}
{2,6,7,8, 9}	{1,2,3,4,5, 6,7,8,9}	3	9	{1,2,3,4,5, 6,7,8,9}	5	1	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}
{2,6,7,8, 9}	4	{1,2,3,4,5, 6,7,8,9}	3	{1,2,3,4,5, 6,7,8,9}	2	{1,2,3,4,5, 6,7,8,9}	5	{1,2,3,4,5, 6,7,8,9}
1	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	3	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	2
5	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	2	{1,2,3,4,5, 6,7,8,9}	4	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	9
{2,6,7,8, 9}	3	8	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	{1,2,3,4,5, 6,7,8,9}	4	6	{1,2,3,4,5, 6,7,8,9}

Propagate AllDiff on 3 × 3 square

...And so on:

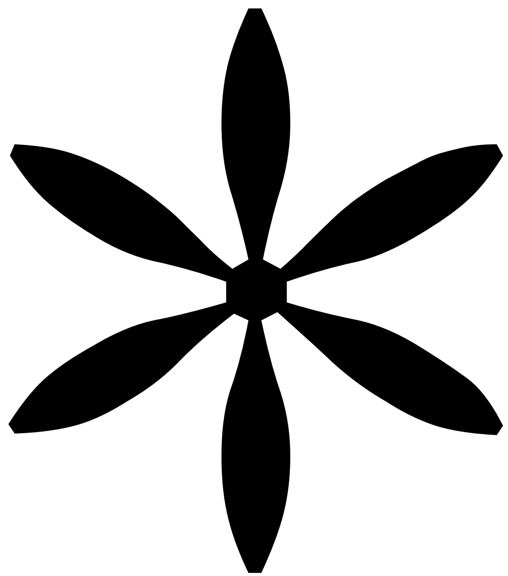
{7}	2	6	{4}	{9}	{3}	8	1	{5}
3	{1}	{5}	7	{2}	8	{9}	{4}	6
4	{8}	{9}	{6}	5	{1}	{2}	{3}	7
{8}	5	{2}	1	{4}	7	{6}	9	{3}
{6}	{7}	3	9	{8}	5	1	{2}	{4}
{9}	4	{1}	3	{6}	2	{7}	5	{8}
1	{9}	{4}	{8}	3	{6}	{5}	{7}	2
5	{6}	{7}	2	{1}	4	{3}	{8}	9
{2}	3	8	{5}	{7}	{9}	4	6	{1}

Backtracking

- Generally, it doesn’t go as well as this
- Search will often involve backtracking
  - i.e. caching current state & making guess
  - Restoring cached state if guess fails
- No backtracking necessary this time
- Backtracking fast is critical in general because problems of this type can require a lot of search to solve.

Constraints & Symmetry

- A familiar everyday concept:



- A **structure-preserving transformation**.
- Constraint problems often exhibit symmetry. Exploiting this symmetry is the subject of our inter-disciplinary research with mathematics.

Example: 4-queens Puzzle

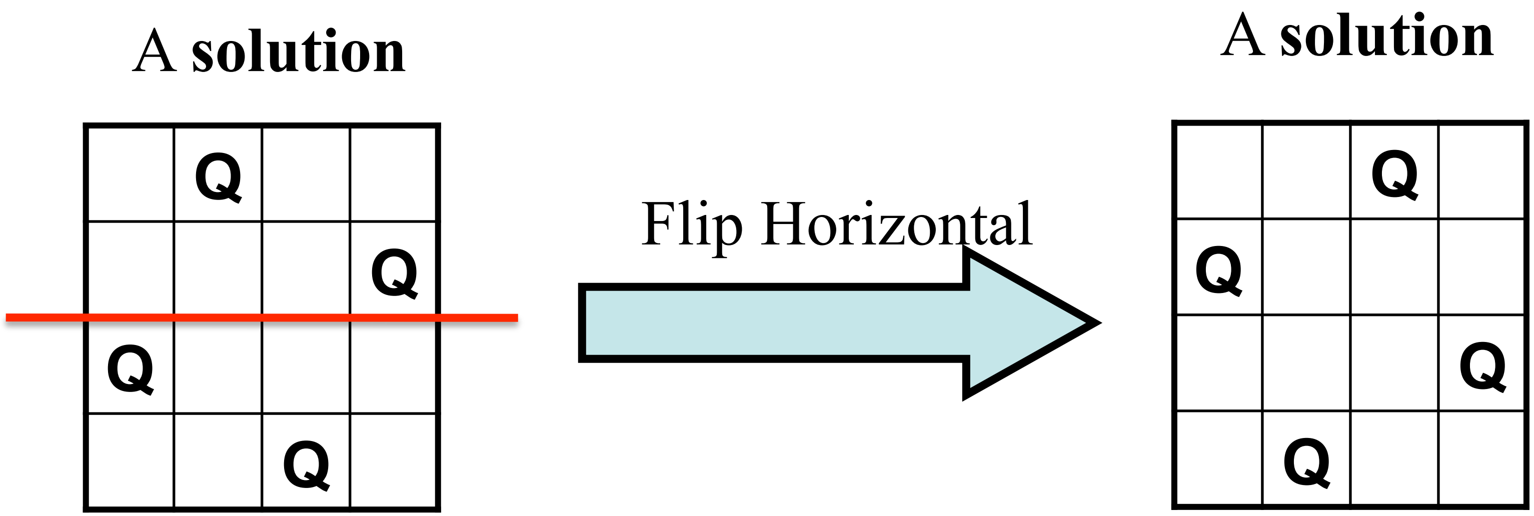
- Place 4 queens on a 4 x 4 chess board such that no pair of queens attack each other.

A **solution**

	Q		
			Q
Q			
		Q	

Example: 4-queens Puzzle

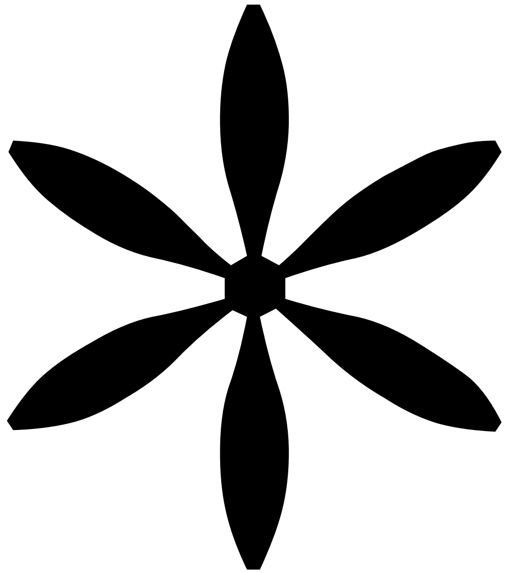
- Place 4 queens on a 4 x 4 chess board such that no pair of queens attack each other.
- Now consider what happens when we flip the chess board horizontally.



- Symmetry partitions solutions into **equivalence classes**.

Exploiting Symmetry

- We can exploit symmetry to **reduce search**.
- Restrict the search to finding one (or a reduced number) of the solutions in each equivalence class.



We want your problems

- If you have a problem that you think we could help to solve, please talk to us!