

# GENERALIZED PATTERN AVOIDANCE, $\beta(1,0)$ -TREES, AND TWO-STACK SORTABLE PERMUTATIONS

ANDERS CLAESSION, SERGEY KITAEV, AND EINAR STEINGRÍMSSON

ABSTRACT. We give a bijection between  $\beta(1,0)$ -trees on  $n + 1$  nodes and length  $n$  permutations avoiding the two generalized patterns 3-1-4-2 and 2-41-3. We show that under this bijection an 8-tuple of statistics on the trees is mapped to an 8-tuple of statistics on the avoiders, giving their equidistribution. For example, the number of leaves is mapped to the number of descents plus one, and root label is mapped to the number of left-to-right minima. One of the other statistics involves an involution on  $\beta(1,0)$ -trees (that does not seem to appear in the literature) with several nice properties: the root label is mapped to the length of the rightmost path and vice versa; leaves are mapped to non-leaves and vice versa; irreducible tree components in one sense become those in another sense and vice versa.

It is known that two-stack sortable permutations of length  $n$  with  $k$  descents are equinumerous with  $\beta(1,0)$ -trees on  $n + 1$  nodes with  $k + 1$  leaves. Thus our bijection also establishes that two-stack sortable permutations of length  $n$  with  $k$  descents are equinumerous with {3-1-4-2, 2-41-3}-avoiding permutations of length  $n$  with  $k$  descents.

It turns out that avoiders are much more symmetric in many respects than two-stack sortable permutations. For example, avoiders are closed under the composition of reverse and compliment or, which is a less trivial fact, under the composition of inverse and reverse, while none of the trivial bijections, or composition of those, preserves the property of being a two-stack sortable permutation. However, we conjecture the existence of a bijection between avoiders and two-stack sortable permutations preserving at least four (natural) permutation statistics.