# Avoidance of Partially Ordered Generalized Patterns of the form $k$-$\sigma$-$k$

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**Abstract**

Sergey Kitaev [Partially Ordered Generalized Patterns, *Discrete Mathematics* 298 (2005), 212-229] and Sergi Elizalde [Asymptotic enumeration of permutations avoiding generalized patterns, *Advances in Applied Mathematics* 36 (2006), 138-155] have independently shown that the exponential generating function for permutations avoiding the generalized pattern $\sigma$-$k$, where $\sigma$ is a pattern without dashes and $k$ is one greater than the biggest element in $\sigma$, is determined by the exponential generating function for permutations avoiding $\sigma$.

We show that this also holds for permutations avoiding all the generalized patterns $\sigma_1$-$k_1$, $\ldots$, $\sigma_n$-$k_n$, where $\sigma_1, \ldots, \sigma_n$ are patterns without dashes and $k_i$ is one greater than the biggest element in $\sigma_i$. Similarly the exponential generating function for permutations avoiding the partially ordered generalized patterns $k_1$-$\sigma_1$-$k_1$, $\ldots$, $k_n$-$\sigma_n$-$k_n$ can be determined from the exponential generating function for permutations avoiding the generalized patterns $\sigma_1, \ldots, \sigma_n$, where $\sigma_1, \ldots, \sigma_n$ are patterns without dashes and $k_i$ is one greater than the biggest element in $\sigma_i$. Since $k$ is the greatest element in the pattern $k$-$\sigma$-$k$, avoidance of $k$-$\sigma$-$k$ is equivalent to avoidance of $(k+1)$-$\sigma$-$k$ and $k$-$\sigma$-$(k+1)$.

This can be used to construct a bijection between bi-colored partitions and permutations avoiding the partially ordered generalized pattern 3-12-3 (that is, permutations avoiding both the patterns 3-12-4 and 4-12-3). By using this method twice a closed formula for the exponential generating function for permutations avoiding the partially ordered generalized pattern 3-212-3 can be found.