Partition Statistics and q-Fibonacci Numbers

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Since the time of Percy MacMahon there has been much interest in distributions of statistics on certain combinatorial objects. The study of pattern avoidance has introduced a whole new natural family of sets where we can study these statistics. In this talk, we will discuss the distribution of the rb statistic, introduced by Wachs and White, over the set of layered matchings, M_n^ℓ, of the set $[n] = \{1, 2, \ldots, n\}$. The set of layered matchings are those set partitions, which avoid the set of patterns $\{13/2, 123\}$. It happens that the distribution of this statistic produces a nice q-analogue, $F_n(q)$ of the Fibonacci numbers. The $F_n(q)$ are closely related to q-Fibonacci numbers studied by both Carlitz and Cigler. This new perspective on these numbers allows us to produce nearly 30 different analogues of our favorite Fibonacci identities. We will focus on the identity $F_n = \sum_{2k \leq n} \binom{n-k}{k}$ and use bijections with integer partitions to prove the analogue,

$$F_n(q) = \sum_{2k \leq n} q^{\binom{n}{2} - k(n-k)} \left[\begin{array}{c} n-k \\ k \end{array} \right].$$