SOME MORE PROPERTIES OF PERMUTATION TABLEAUX

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1. Abstract

This talk extends the permutation tableau results presented at PP'2006. In particular, it was shown via 0-hinge tableaux that the number of essential 1s in the tableaux and the number of cycles in permutations are equidistributed statistics. Here we give a different approach to that result via the following theorem.

Theorem 1.1. *Let $\pi \in S_n$ be a permutation with k weak excedances and let T be its 1-hinge tableau whose 1s and 0s form a Ferrers board λ. Then, for each partition $\mu \le \lambda$, π has a unique tableau with at least one 1 in each column such that the 1-hinge rule applies within μ and the 0-hinge rule applies within λ/μ.*

The map $irc = i \circ r \circ c$ (inverse of reversal of complement) is known to preserve almost all alignment and crossing statistics on permutation tableaux except for exchanging statistics A_{NE} and A_{EN}. We show that the tableau of $irc(\pi)$ can be easily computed from the tableau of π for $\pi \in S_n(321)$ and for π whose tableaux have no 2s, and no 0s on the main diagonal.

We also enumerate some restricted sets of permutations whose 1-hinge tableaux have the maximal number of essential 1s. For example, let $M_n(\tau)$ denote the set of permutations in $S_n(\tau)$ whose 1-hinge tableaux have $n-1$ essential 1s (i.e. the maximum number). Then we determine the structure of the these tableaux to prove some enumerative results including the following.

Theorem 1.2. *The number of permutations in $M_n(132, 231)$ (resp. in $M_n(213, 312)$) whose 1-hinge tableaux have k rows (resp. k columns) is equal to*

$$2^{k-1}\binom{n-k-1}{k-1} - 2^{k-2}\binom{n-k-2}{k-2}.$$

Theorem 1.3. *If $M_n^k(123, 213)$ is the set of permutations in $M_n(123, 213)$ with k nonessential 1s, then*

$$|M_{2n}^k(123, 213)| = a(2n-2, k) + a(2n-3, k),$$
$$|M_{2n+1}^k(123, 213)| = 2a(2n-2, k) + a(2n-3, k),$$

where $a(n, k) = \mathrm{A037027}(n, k)$, the kth entry in row n of the Fibonacci-Pascal triangle.

Theorem 1.4. *Let $M_n(123, 213; k)$ be the set of permutations in $M_n(123, 213)$ that start with the letter k. Then, for $1 \le k \le n \le 2k+3$, we have*

$$|M_n(123, 213; k)| = 2^{n-k}\mathrm{A002965}(2k-n-3).$$

Theorem 1.5.

$$|M_n(132, 213)| = \begin{cases} a(n) = 2\cdot 3^{\frac{n-2}{2}} - 1 & \text{if } n \text{ is even,} \\ a(n) = 3^{\frac{n-1}{2}-1} & \text{if } n \text{ is odd.} \end{cases}$$

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