Osculating Paths and Oscillating Tableaux

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In this talk, the combinatorics of certain osculating lattice paths and oscillating tableaux will be discussed, and it will be shown that there is a natural relationship between these two combinatorial objects. Full details of this work, and further results, can be found in arXiv:math.CO/0701755.

More specifically, path tuples will be considered in which each path has a fixed start and end point on respectively the lower and right boundaries of a rectangle in $\mathbb{Z}^2$, each path can take only unit steps rightwards or upwards, and two different paths are permitted to share lattice points, but not to cross or share lattice edges. Such path tuples include cases which correspond to alternating sign matrices and various subclasses thereof. Referring to points of the rectangle through which no or two paths pass as vacancies or osculations respectively, it will be shown that there exist bijections which map each path tuple P with l vacancies and osculations to a pair (t, η), where η is an oscillating tableau of length l (i.e., a sequence of $l+1$ partitions, starting with the empty partition, in which the Young diagrams of successive partitions differ by a single square), and t is a certain, compatible sequence of l weakly-increasing positive integers. In these bijections, which can be regarded as generalizations of well-known bijections between certain tuples of nonintersecting lattice paths and semistandard Young tableaux, each vacancy or osculation of P corresponds to a partition in η whose Young diagram is obtained from that of its predecessor by respectively the addition or deletion of a square.

In describing these bijections, it will be useful to define the *profile* of an oscillating tableau $\eta = (\eta_0, \eta_1, \ldots, \eta_l)$ to be the sequence $(j_1 - i_1, \ldots, j_l - i_l)$, where (i_k, j_k) is the position of the square by which the Young diagrams of η_k and η_{k-1} differ. For example, the profile of $(\emptyset, (1), (2), (3), (3,1), (2,1), (1,1))$ is $(0, 1, 2, -1, 2, 1)$, and it can be seen that any oscillating tableau is uniquely determined by its profile. In this talk, some properties of these sequences of integers, and the patterns they can form, will also be discussed.