ASPECTS OF SEPARABILITY

MICHAEL ALBERT

A *separable permutation* is one which contains neither the pattern 2413, nor the pattern 3142. The class, $\mathcal{SEP}$, of all separable permutations is enumerated by the large Schroeder numbers and has growth rate $3+2\sqrt{2}$. It contains no simple permutations other than 12 and 21 and as a consequence has a very strong structure theory. We will illustrate some aspects of this theory by discussing the following results:

- The class $\mathcal{SEP}$ is *growth rate critical*. That is, if $\mathcal{C}$ is any proper subclass of $\mathcal{SEP}$, then
$$\limsup_{n\to\infty} |\mathcal{C} \cap \mathcal{S}_n|^{1/n} < 3 + 2\sqrt{2}.$$
- Let $\mathcal{C}$ be any subclass of the separable permutations, and let $c(t)$ be its generating function. Then, the degree of c over $\mathbb{Q}(t)$ is a power of 2.
- Let $(\pi_n)_{n\geq 1}$ be the sequence of permutations:
$$132, 4132, 15243, 615243, 1726354, \ldots$$
where π_n is obtained from π_{n-1} by preceding its pattern with either a new maximum or a new minimum according as whether the parity of n is even or odd. Let $\mathcal{C}_n$ be the subclass of $\mathcal{SEP}$ consisting of those separable permutations that also avoid π_n, and let $c_n(t)$ be its generating function. Then, the degree of c_n over $\mathbb{Q}(t)$ is exactly 2^n.

Department of Computer Science, University of Otago

E-mail address: `malbert@cs.otago.ac.nz`